

Generalized Schwinger proper time method for Dirac operator with dynamical chiral symmetry breaking

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Abstract

Schwinger proper time method is generalized for the calculation of real part of determinant and coincidence limit of inverse for Dirac operator with dynamical chiral symmetry breaking caused by momentum dependent fermion self energy $\Sigma(k^2)$. The obtained series generalizes the heat kernel expansion for hard fermion mass.

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I. INTRODUCTION

In a renormalizable quantum field theory at 4-dimensional Euclidean space time, the local interaction of a fermion system with external fields is bilinear in fermion fields and realized in terms of Dirac operator D ,

$$D \equiv \not{\nabla} - s + ip\gamma_5 \quad \nabla_\mu \equiv \partial_\mu - iv_\mu - ia_\mu\gamma_5 = -\nabla_\mu^\dagger, \quad (1)$$

where s , p , v_μ , a_μ are hermitian fields fermion coupled with and γ matrix is hermitian. We have decomposed these fields in terms their Lorentz structures ¹. In terms of Dirac spinor ψ and $\bar{\psi}$, the action of renormalizable interaction is

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¹In general, there should be tensor field, but for simplicity of discussion, in this paper, we donot consider the tensor field.

$$\int d^4x \bar{\psi} D \psi \quad (2)$$

This action is invariant under following local chiral transformations

$$\begin{aligned} \psi(x) &\rightarrow \psi'(x) = [V_L(x)P_L + V_R(x)P_R]\psi(x) \\ J(x) &\rightarrow J'(x) = [V_L(x)P_R + V_R(x)P_L][J(x) + i\boldsymbol{\not{D}}_x][V_L^\dagger(x)P_L + V_R^\dagger(x)P_R] \end{aligned} \quad (3)$$

with $V_L(x)$ and $V_R(x)$ be left and right chiral rotation matrices and project operator $P_L \equiv \frac{1}{2}(1 \pm \gamma_5)$. The external field $J(x)$ is defined as

$$J(x) = -i\not{p}(x) - i\not{d}(x)\gamma_5 - s(x) + ip(x)\gamma_5 \quad (4)$$

Suppose we have N_f Dirac spinors in our fermion system, this local chiral symmetry then is $U_L(N_f) \otimes U_R(N_f)$.

The contribution of action (2) to a path integral of 4- dimension space time quantum field theory can be written as

$$\int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{\int d^4x [\bar{\psi} D \psi + \bar{I} \psi + \bar{\psi} I]} = e^{\text{Tr} \ln D - \int d^4x \int d^4y \bar{I}(x) D^{-1}(x,y) I(y)} \quad (5)$$

where \bar{I} and I are external sources of the fermion system. Due to bilinear property of fermion interaction, we have integrated out fermions in the path integral (5) and found that the contribution of fermion to the path integral is realized through fermion determinant $\text{Tr} \ln D$ and propagator $D^{-1}(x, y) = \langle x | D^{-1} | y \rangle$ in presence of external fields. Once these two objects are obtained, the contribution of fermion system to a 4-dim space time renormalizable field theory is known.

In practice, for the physical needs and regularization of infrared divergence, a constant bare mass m is usually added into the theory which is equivalent to extract out from the scalar field s a condensation part $s \rightarrow s - m$. With this quark mass term, interaction action (2) become

$$\int d^4x \bar{\psi} (D + m) \psi \quad (6)$$

and two ingredients of fermion system in (5) become logarithm of fermion determinant $\text{Tr} \ln(D + m)$ and fermion propagator $(D + m)^{-1}(x, y)$.

Adding in quark mass term explicitly violate original chiral symmetry from $U_L(N_f) \otimes U_R(N_f)$ to $U_V(N_f)$, whose transformation is in fact requiring the original left handed and right handed rotations be same $V_L(x) = V_R(x) \equiv h^\dagger(x)$. In the literature, there is discussion on how to generalize the constant fermion mass to a matrix [1] to respect further breaking

of the $U_V(N_f)$ symmetry in the fermion system. In this work, we are interested in the generalization in another direction– the case of dynamical violating the chiral symmetry. Within context of dynamical chiral symmetry breaking (DCSB), the most general situation is not the appearance of a hard fermion mass m , but a momentum k^2 dependent fermion self energy $\Sigma(k^2)$. In fact, in the quantum field theory, even the chiral symmetry is broken by a momentum independent hard mass term at leading order of calculation, include in high order quantum corrections, the hard mass will be replaced by momentum dependent fermion self energy. Taking hard fermion mass often cause extra ultraviolet divergence, instead, momentum dependent fermion self energy usually will suppress the ultraviolet divergence. For example, in the case of massless QCD, the fermion self energy damping at least as $1/p^2$ at ultraviolet momentum region, if we take a hard fermion mass to substitute self energy, we will over estimate its contribution to physics and cause extra divergence which usually lead many physical fine tuning problems. To respect the momentum dependence of fermion self energy in the theory, in this paper, we generalize the conventional Schwinger proper time formulation [2] to deal with corresponding fermion determinant and propagator. For simplicity and as the first step of research in this direction we only discuss the real part of fermion determinant known conventionally as effective action and part of coincidence limit of fermion propagator related to fermion pair condensation. The imaginary part of fermion determinant related to anomaly and remaining nonlocal part of fermion propagator will be discussed else where.

This paper is organized as follows: In section II, we review the conventional Schwinger proper time formulation for the real part of fermion determinant and coincidence limit of fermion propagator. In section III, we discuss how to generalize the formulation from a hard mass case to momentum dependent dynamical fermion self energy. Section IV is responsible for the calculation of the real part of fermion determinant and section V is responsible for the calculation of the coincidence limit of fermion propagator in which the tensor structure is ignored in our calculation. Section VI is summary and discussion.

II. REVIEW OF SCHWINGER PROPER TIME FORMULATION FOR DIRAC OPERATOR WITH A HARD FERMION MASS

The real part of fermion determinant in conventional Schwinger proper time formulation [2] is

$$\text{Re ln Det}(D + m) = \frac{1}{2} \text{Tr ln}[(D^\dagger + m)(D + m)]$$

$$= \frac{1}{2} \lim_{\Lambda \rightarrow \infty} \left[-\gamma + \ln \Lambda^2 - \int d^4x \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{d\tau}{\tau} \text{tr} e^{-m^2\tau} \langle x | e^{-\tau(E-\nabla^2)} | x \rangle \right] \quad (7)$$

where tr is trace for internal symmetry indices, such as spinor, flavor indices, etc. And

$$E - \nabla^2 = D^\dagger D + D^\dagger m + mD \quad (8)$$

with

$$\begin{aligned} (\nabla)^\dagger &\equiv -\not{\partial} + i\not{\psi} - i\not{\phi} \gamma_5 & D^\dagger &= \nabla^\dagger - s - ip\gamma_5 \\ E &= -2ms - 2im\not{\phi} \gamma_5 + \frac{i}{4}[\gamma^\mu, \gamma^\nu] \mathcal{R}_{\mu\nu} + \gamma_\mu d^\mu (s - ip\gamma_5) + i\gamma^\mu [a_\mu \gamma_5 (s - ip\gamma_5) + (s - ip\gamma_5) a_\mu \gamma_5] \\ &\quad + s^2 + p^2 - [s, p] i\gamma_5 \end{aligned} \quad (9)$$

$$\begin{aligned} \mathcal{R}_{\mu\nu} &\equiv i[\nabla_\mu, \nabla_\nu] = [d_\mu a_\nu - d_\nu a_\mu] \gamma_5 + V_{\mu\nu} - i[a_\mu, a_\nu] & V_{\mu\nu} &= \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu] \\ d_\mu f &= \partial_\mu f - i[v_\mu, f] & d_\mu(fg) &= (d_\mu f)g + f(d_\mu g) \end{aligned}$$

In getting result (7), we have used following relation for a matrix $A \equiv (D^\dagger + m)(D + m) = E - \nabla^2 + m^2$,

$$\int_{\frac{1}{\Lambda^2}}^{\infty} \frac{e^{-A\tau}}{\tau} d\tau = -\text{Ei}\left(-\frac{A}{\Lambda^2}\right) = -\gamma - \ln\left(\frac{A}{\Lambda^2}\right) - \sum_{n=1}^{\infty} \frac{(-1)^n}{n!n} \left(\frac{A}{\Lambda^2}\right)^n \quad (10)$$

In Schwinger proper time formulation, the regularization is done at lower limit of proper time integration τ . Since proper time is a parameter irrelevant to symmetry, this regularization has the feature of keeping symmetry of the system.

With help of standard Seely-DeWitt expansion [3],

$$\begin{aligned} \langle x | e^{-\tau(E-\nabla^2)} | x \rangle &= \frac{1}{16\pi^2} \left[\frac{1}{\tau^2} - \frac{E}{\tau} + \frac{1}{2}E^2 - \frac{1}{6}[\nabla_\mu, [\nabla^\mu, E]] - \frac{1}{12}\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} - \frac{\tau}{6}E^3 \right. \\ &\quad \left. + \frac{\tau}{12}\{E[\nabla^\mu, [\nabla_\mu, E]] + [\nabla^\mu, [\nabla_\mu, E]]E + [\nabla^\mu, E][\nabla_\mu, E]\} + \frac{\tau^2}{24}E^4 + \dots \right], \end{aligned} \quad (11)$$

the proper time integration of τ in (7) can be finished, substitute (9) into result formulae, we get the expansion of $\text{Re} \ln \det(D + m)$ with hard fermion mass m ,

$$\begin{aligned} &\text{Re} \ln \det(D + m) \\ &= \text{Re} \ln \det(\not{\partial} + m) - \frac{1}{32\pi^2} \lim_{\Lambda \rightarrow \infty} \int d^4x \text{tr}_f \left[8m[\Lambda^2 + m^2(\ln \frac{m^2}{\Lambda^2} + \gamma - 1)]s \right. \\ &\quad - 8m^2(\ln \frac{m^2}{\Lambda^2} + \gamma)a^2 - \frac{4}{3}[d_\mu a^\mu]^2 - \frac{2}{3}(\ln \frac{m^2}{\Lambda^2} + \gamma + 1)(d_\mu a_\nu - d_\nu a_\mu)(d^\mu a^\nu - d^\nu a^\mu) \\ &\quad - [\frac{4}{3}(\ln \frac{m^2}{\Lambda^2} + \gamma) + \frac{16}{3}]a^4 + [\frac{4}{3}(\ln \frac{m^2}{\Lambda^2} + \gamma) + \frac{8}{3}]a_\mu a_\nu a^\mu a^\nu - 4[\Lambda^2 + m^2(3\ln \frac{m^2}{\Lambda^2} + 3\gamma - 1)]s^2 \\ &\quad \left. - 4[\Lambda^2 + m^2(\ln \frac{m^2}{\Lambda^2} + \gamma - 1)]p^2 + (16\ln \frac{m^2}{\Lambda^2} + 16\gamma + 16)msa^2 - \frac{2}{3}(\ln \frac{m^2}{\Lambda^2} + \gamma)V_{\mu\nu}V^{\mu\nu} \right] \end{aligned}$$

$$\begin{aligned}
& +i\left[\frac{8}{3}\left(\ln\frac{m^2}{\Lambda^2} + \gamma\right) + \frac{16}{3}\right]a^\mu a^\nu V_{\mu\nu} + 8m\left(\ln\frac{m^2}{\Lambda^2} + \gamma\right)pd^\mu a_\mu + \epsilon^{\sigma\rho\mu\nu}[-2\left(\ln\frac{m^2}{\Lambda^2} + \gamma\right)V_{\sigma\rho}d_\mu a_\nu \\
& + 4i\left(\ln\frac{m^2}{\Lambda^2} + \gamma + 2\right)a_\sigma a_\rho d_\mu a_\nu] + O(p^6) \quad , \quad (12)
\end{aligned}$$

where tr_f is the trace for flavor index and we have isolated out the external fields independent term

$$\begin{aligned}
\text{Re ln Det}(\not{D} + m) &= -\frac{1}{2} \lim_{\Lambda \rightarrow \infty} \int d^4x \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{d\tau}{\tau} \text{tr} e^{-m^2\tau} \langle x | e^{\tau \not{D}^2} | x \rangle \\
&= -\frac{1}{32\pi^2} \lim_{\Lambda \rightarrow \infty} \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{d\tau}{\tau^3} e^{-m^2\tau} \int d^4x \text{tr} 1 \\
&= -\frac{1}{32\pi^2} \lim_{\Lambda \rightarrow \infty} [2\Lambda^4 - 4\Lambda^2 m^2 - m^4(2\ln\frac{m^2}{\Lambda^2} + 2\gamma - 1)] \int d^4x \text{tr} 1 \quad (13)
\end{aligned}$$

and the expansion terms in (12) are arranged according to its momentum power in which ∂_μ , v_μ , a_μ are treated as order p and s , p as order p^2 [4]. Hermitian matrix γ_5 is defined as $\gamma_5 = -\gamma^0\gamma^1\gamma^2\gamma^3$ which leads relations $\text{tr}_l(\gamma_5\gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho) = -4\epsilon^{\mu\nu\sigma\rho}$ and $\gamma^\mu\gamma^\sigma\gamma^\rho = \gamma^\mu g^{\sigma\rho} - \gamma^\sigma g^{\mu\rho} + \gamma^\rho g^{\mu\sigma} - \epsilon_\nu^{\mu\sigma\rho}\gamma^\nu\gamma_5$.

There are two ways to calculate coincidence limit of fermion propagator, one is direct perform functional differential with respect to corresponding external fields and the other is taking similar computation procedure as that for fermion determinant. We will see two methods give exactly same results.

First we consider the functional differential of fermion determinant. Use identity

$$\begin{aligned}
\frac{\delta \text{Tr ln}(D + m)}{\delta J^{\sigma\rho}(x)} &= \int d^4y d^4z (D + m)^{-1, \rho'\sigma'}(y, z) \frac{\delta D^{\sigma'\rho'}(z, y)}{\delta J^{\sigma\rho}(x)} = (D + m)^{-1, \rho\sigma}(x, x) \\
\frac{\delta \text{Tr ln}(D^\dagger + m)}{\delta J^{\dagger\sigma\rho}(x)} &= \int d^4y d^4z (D^\dagger + m)^{-1, \rho'\sigma'}(y, z) \frac{\delta D^{\dagger\sigma'\rho'}(z, y)}{\delta J^{\dagger\sigma\rho}(x)} = (D^\dagger + m)^{-1, \rho\sigma}(x, x) \quad (14)
\end{aligned}$$

where $\sigma = (a\xi)$, $\rho = (b\zeta)$ (a, b are flavor indices, ξ, ζ are Lorentz indices). The scalar part of coincidence limit of fermion propagator then is

$$\begin{aligned}
& -\frac{\delta \text{ReTr ln}(D + m)}{\delta s^{ab}(x)} = -\frac{1}{2} \left[\frac{\delta \text{Tr ln}(D + m)}{\delta s^{ab}(x)} + \frac{\delta \text{Tr ln}(D^\dagger + m)}{\delta s^{ab}(x)} \right] \\
&= -\frac{1}{2} \int d^4y \left[\frac{\delta \text{Tr ln}(D + m)}{\delta J^{\sigma\rho}(y)} \frac{\delta J^{\sigma\rho}(y)}{\delta s^{ab}(x)} + \frac{\delta \text{Tr ln}(D^\dagger + m)}{\delta J^{\dagger\sigma\rho}(y)} \frac{\delta J^{\dagger\sigma\rho}(y)}{\delta s^{ab}(x)} \right] \\
&= \frac{1}{2} (1)^{\xi\zeta} [(D + m)^{-1, (b\zeta)(a\xi)}(x, x) + (D^\dagger + m)^{-1, (b\zeta)(a\xi)}(x, x)] \\
&= \frac{1}{4\pi^2} \lim_{\Lambda \rightarrow \infty} \left[m[\Lambda^2 + m^2(\ln\frac{m^2}{\Lambda^2} + \gamma - 1)]\delta^{ba} - [\Lambda^2 + m^2(3\ln\frac{m^2}{\Lambda^2} + 3\gamma - 1)]s^{ba} \right. \\
&\quad \left. + (2\ln\frac{m^2}{\Lambda^2} + 2\gamma + 2)m(a^2)^{ba} \right] + O(p^4) \quad , \quad (15)
\end{aligned}$$

similarly the pseudo scalar part is

$$\begin{aligned}
-i \frac{\delta \text{ReTr} \ln(D+m)}{\delta p^{ab}(x)} &= -\frac{i}{2} \left[\frac{\delta \text{Tr} \ln(D+m)}{\delta p^{ab}(x)} + \frac{\delta \text{Tr} \ln(D^\dagger+m)}{\delta p^{ab}(x)} \right] \\
&= -\frac{i}{2} \int d^4 y \left[\frac{\delta \text{Tr} \ln(D+m)}{\delta J^{\sigma\rho}(y)} \frac{\delta J^{\sigma\rho}(y)}{\delta p^{ab}(x)} + \frac{\delta \text{Tr} \ln(D^\dagger+m)}{\delta J^{\dagger\sigma\rho}(y)} \frac{\delta J^{\dagger\sigma\rho}(y)}{\delta p^{ab}(x)} \right] \\
&= \frac{1}{2} (\gamma_5)^{\xi\zeta} [(D+m)^{-1, (b\zeta)(a\xi)}(x, x) - (D^\dagger+m)^{-1, (b\zeta)(a\xi)}(x, x)] \\
&= \frac{1}{4\pi^2} \lim_{\Lambda \rightarrow \infty} \left[-i[\Lambda^2 + m^2(\ln \frac{m^2}{\Lambda^2} + \gamma - 1)]p^{ba} + im(\ln \frac{m^2}{\Lambda^2} + \gamma)(d^\mu a_\mu)^{ba} \right] + O(p^4) \quad , \quad (16)
\end{aligned}$$

the vector part is

$$\begin{aligned}
i \frac{\delta \text{ReTr} \ln(D+m)}{\delta v_\mu^{ab}(x)} &= \frac{i}{2} \left[\frac{\delta \text{Tr} \ln(D+m)}{\delta v_\mu^{ab}(x)} + \frac{\delta \text{Tr} \ln(D^\dagger+m)}{\delta v_\mu^{ab}(x)} \right] \\
&= \frac{i}{2} \int d^4 y \left[\frac{\delta \text{Tr} \ln(D+m)}{\delta J^{\sigma\rho}(y)} \frac{\delta J^{\sigma\rho}(y)}{\delta v_\mu^{ab}(x)} + \frac{\delta \text{Tr} \ln(D^\dagger+m)}{\delta J^{\dagger\sigma\rho}(y)} \frac{\delta J^{\dagger\sigma\rho}(y)}{\delta v_\mu^{ab}(x)} \right] \\
&= \frac{1}{2} (\gamma_\mu)^{\xi\zeta} [(D+m)^{-1, (b\zeta)(a\xi)}(x, x) - (D^\dagger+m)^{-1, (b\zeta)(a\xi)}(x, x)] \\
&= -\frac{i}{32\pi^2} \lim_{\Lambda \rightarrow \infty} \left[-\frac{8i}{3} [d_\nu a^\nu, a^\mu] - \frac{8i}{3} (\ln \frac{m^2}{\Lambda^2} + \gamma + 1) [(d^\mu a^\nu - d^\nu a^\mu), a_\nu] - \frac{8}{3} (\ln \frac{m^2}{\Lambda^2} + \gamma) d^\nu V^{\mu\nu} \right. \\
&\quad + i \left[\frac{8}{3} (\ln \frac{m^2}{\Lambda^2} + \gamma) + \frac{16}{3} \right] d_\nu ([a^\mu, a^\nu]) + 8im(\ln \frac{m^2}{\Lambda^2} + \gamma) [p, a^\mu] + 2i\epsilon^{\nu\sigma\rho\mu} (\ln \frac{m^2}{\Lambda^2} + \gamma) \left(2id^\mu d_\sigma a_\rho \right. \\
&\quad \left. \left. + [a^\mu, V_{\sigma\rho}] \right) \right]^{ba} + O(p^5) \quad , \quad (17)
\end{aligned}$$

the axial vector part is

$$\begin{aligned}
-i \frac{\delta \text{ReTr} \ln(D+m)}{\delta a_\mu^{ab}(x)} &= -\frac{i}{2} \left[\frac{\delta \text{Tr} \ln(D+m)}{\delta a_\mu^{ab}(x)} + \frac{\delta \text{Tr} \ln(D^\dagger+m)}{\delta a_\mu^{ab}(x)} \right] \\
&= -\frac{i}{2} \int d^4 y \left[\frac{\delta \text{Tr} \ln(D+m)}{\delta J^{\sigma\rho}(y)} \frac{\delta J^{\sigma\rho}(y)}{\delta a_\mu^{ab}(x)} + \frac{\delta \text{Tr} \ln(D^\dagger+m)}{\delta J^{\dagger\sigma\rho}(y)} \frac{\delta J^{\dagger\sigma\rho}(y)}{\delta a_\mu^{ab}(x)} \right] \\
&= \frac{1}{2} (\gamma_5 \gamma_\mu)^{\xi\zeta} [(D+m)^{-1, (b\zeta)(a\xi)}(x, x) + (D^\dagger+m)^{-1, (b\zeta)(a\xi)}(x, x)] \\
&= \frac{i}{4\pi^2} \lim_{\Lambda \rightarrow \infty} \left[-2m^2(\ln \frac{m^2}{\Lambda^2} + \gamma) a^\mu + \frac{1}{3} [d^\mu d^\nu a_\nu] - \frac{1}{3} (\ln \frac{m^2}{\Lambda^2} + \gamma + 1) d^\nu (d^\mu a_\nu - d_\nu a^\mu) \right. \\
&\quad - \left[\frac{1}{3} (\ln \frac{m^2}{\Lambda^2} + \gamma) + \frac{4}{3} \right] (a^2 a^\mu + a^\mu a^2) + \left[\frac{2}{3} (\ln \frac{m^2}{\Lambda^2} + \gamma) + \frac{4}{3} \right] a_\nu a^\mu a^\nu \\
&\quad + 2(\ln \frac{m^2}{\Lambda^2} + \gamma + 1) m (s a^\mu + a^\mu s) + i \left[\frac{1}{3} (\ln \frac{m^2}{\Lambda^2} + \gamma) + \frac{2}{3} \right] (a_\nu V^{\mu\nu} - V^{\mu\nu} a_\nu) \\
&\quad \left. - m(\ln \frac{m^2}{\Lambda^2} + \gamma) d^\mu p - \frac{1}{4} (\ln \frac{m^2}{\Lambda^2} + \gamma) d^\mu V_{\sigma\rho} \epsilon^{\nu\sigma\rho}{}_\mu \right]^{ba} + O(p^5) \quad . \quad (18)
\end{aligned}$$

Since tensor fields are not introduced into the theory, we cannot obtain the tensor part by performing functional differential.

We then take the second method, write propagator in presence of external fields as

$$\begin{aligned}
& (D + m)^{-1}(x, x) \\
&= \left[[(D^\dagger + m)(D + m)]^{-1}(D^\dagger + m) \right](x, x) \\
&= \lim_{\Lambda \rightarrow \infty} \int_{\frac{1}{\Lambda^2}}^{\infty} d\tau e^{-m^2\tau} \langle x | e^{-\tau(E - \nabla^2)} (D^\dagger + m) | x \rangle \\
&= \lim_{\Lambda \rightarrow \infty} \int_{\frac{1}{\Lambda^2}}^{\infty} d\tau e^{-m^2\tau} \left[-\langle x | e^{-\tau(E - \nabla^2)} \not{\nabla} | x \rangle + \langle x | e^{-\tau(E - \nabla^2)} | x \rangle [m - s(x) - ip(x)\gamma_5 \right. \\
&\quad \left. - 2i\not{p}(x)\gamma_5] \right] \tag{19}
\end{aligned}$$

Similar as expansion (11), we have

$$\langle x | e^{-\tau(E - \nabla^2)} \nabla_\mu | x \rangle = \frac{1}{16\pi^2} \left[\frac{i}{6\tau} [\nabla^\nu, \mathcal{R}_{\nu\mu}] + \frac{1}{2\tau} [\nabla_\mu, E] - \frac{1}{3} E [\nabla_\mu, E] - \frac{1}{6} [\nabla_\mu, E] E \right] + \dots \tag{20}$$

Substitute (20) back to (19) and finish the integration of τ , we obtain the expansion of $(D + m)^{-1}(x, x)$ with hard fermion mass m ,

$$\begin{aligned}
& (D + m)^{-1}(x, x) \\
&= \frac{1}{16\pi^2} \lim_{\Lambda \rightarrow \infty} \left\{ [\Lambda^2 + m^2(\ln \frac{m^2}{\Lambda^2} + \gamma - 1)](m - ip\gamma_5) - 2im^2(\ln \frac{m^2}{\Lambda^2} + \gamma)\not{p}\gamma_5 \right. \\
&\quad - [\Lambda^2 + m^2(3\ln \frac{m^2}{\Lambda^2} + 3\gamma - 1)]s + im(\ln \frac{m^2}{\Lambda^2} + \gamma)d^\mu a_\mu \gamma_5 + 2m(\ln \frac{m^2}{\Lambda^2} + \gamma + 1)a^2 \\
&\quad + \left[\frac{i}{3}(\ln \frac{m^2}{\Lambda^2} + \gamma)d^\nu V_{\mu\nu} + m(\ln \frac{m^2}{\Lambda^2} + \gamma)(pa^\mu - a^\mu p) - \frac{1}{3}[(d_\nu a^\nu)a^\mu - a^\mu(d_\nu a^\nu)] \right. \\
&\quad + \frac{1}{3}(\ln \frac{m^2}{\Lambda^2} + \gamma + 1)[a_\nu(d^\mu a^\nu - d^\nu a^\mu) - (d^\mu a^\nu - d^\nu a^\mu)a_\nu] \\
&\quad + \frac{1}{3}(\ln \frac{m^2}{\Lambda^2} + \gamma + 2)d^\nu(a_\mu a_\nu - a_\nu a_\mu) + \epsilon_\mu^{\sigma\rho\nu} \left[\frac{i}{2}(\ln \frac{m^2}{\Lambda^2} + \gamma)d^\nu d_\sigma a_\rho \right. \\
&\quad + \frac{1}{4}(\ln \frac{m^2}{\Lambda^2} + \gamma - 2)a^\nu V_{\sigma\rho} - \frac{1}{4}(\ln \frac{m^2}{\Lambda^2} + \gamma + 2)V_{\sigma\rho}a^\nu + \frac{2i}{3}a_\sigma a_\rho a^\nu \left. \right] \gamma^\mu + \left[im(\ln \frac{m^2}{\Lambda^2} + \gamma)d^\mu p \right. \\
&\quad - 2im(\ln \frac{m^2}{\Lambda^2} + \gamma + 1)(a^\mu s + sa^\mu) - \frac{i}{3}(\ln \frac{m^2}{\Lambda^2} + \gamma + 1)d^\nu(d_\nu a^\mu - d_\mu a^\nu) \\
&\quad + \frac{1}{3}(\ln \frac{m^2}{\Lambda^2} + \gamma + 2)(a^\nu V_{\mu\nu} - V_{\mu\nu}a^\nu) - \frac{i}{3}d^\mu d^\nu a_\nu + \frac{i}{3}(\ln \frac{m^2}{\Lambda^2} + \gamma + 4)(a^2 a^\mu + a^\mu a^2) \\
&\quad \left. - \frac{2i}{3}(\ln \frac{m^2}{\Lambda^2} + \gamma + 2)a_\nu a^\mu a^\nu - \epsilon_\mu^{\sigma\rho\nu} \left[-\frac{i}{4}(\ln \frac{m^2}{\Lambda^2} + \gamma)d^\nu V_{\sigma\rho} - \frac{1}{3}a_\sigma d^\nu a_\rho + \frac{1}{3}(d_\sigma a_\rho)a^\nu \right] \gamma_5 \gamma^\mu \right\} \\
&\quad + \text{tensor terms} + O(p^4) \tag{21}
\end{aligned}$$

One can easily check that above result can exactly reproduce results (15), (16), (17) and (18). So two kinds calculation give same results for coincidence limit of fermion propagator.

III. GENERALIZED SCHWINGER PROPER TIME FORMULATION FOR DIRAC OPERATOR WITH DYNAMICAL FERMION SELF ENERGY

In quantum field theory (5) without bare fermion mass, the Lagrangian is chiral symmetric. Usually the realization of chiral symmetry of the system is characterized by symmetry order parameter $\langle 0|\bar{\psi}\psi|0\rangle$, if $\langle 0|\bar{\psi}\psi|0\rangle \neq 0$, we say the chiral symmetry is dynamically broken. This happens only when fermion has a nonzero self energy $\Sigma(k^2)$ which is in general real and momentum dependent. The value of $\Sigma(k^2)$ can be determined by solving dynamical equation, such as Schwinger-Dyson equation. If we plan to use a phenomenological Lagrangian to represent this DCSB effect, since momentum dependent fermion self energy $\Sigma(k^2)$ in coordinate space is represented by $\Sigma(-\partial_x^2)\delta(x-y)$, the naive generalization of fermion interaction for explicit chiral symmetry breaking Lagrangian $\bar{\psi}(D+m)\psi$ to DCSB Lagrangian should be $\bar{\psi}[D+\Sigma(-\partial^2)]\psi$. We argue this is not suitable since original $\bar{\psi}(D+m)\psi$ is invariant under $U_V(N_f)$ local symmetry transformation,

$$\begin{aligned}
\psi(x) &\rightarrow \psi'(x) = h^\dagger(x)\psi(x) \\
s(x) &\rightarrow s'(x) = h^\dagger(x)s(x)h(x) \\
p(x) &\rightarrow p'(x) = h^\dagger(x)p(x)h(x) \\
v_\mu(x) &\rightarrow v'_\mu(x) = h^\dagger(x)v_\mu(x)h(x) + h^\dagger(x)[i\partial_\mu h(x)] \\
a_\mu(x) &\rightarrow a'_\mu(x) = h^\dagger(x)a_\mu(x)h(x) \quad ,
\end{aligned} \tag{22}$$

which leads to

$$\begin{aligned}
D_x + m &\rightarrow (D_x + m)' \equiv \not{\partial}_x - i\not{\psi}'(x) - i\not{\phi}'(x)\gamma_5 + m - s'(x) + ip'(x)\gamma_5 \\
&= h^\dagger(x)[\not{\partial}_x - i\not{\psi}(x) - i\not{\phi}(x)\gamma_5 - s(x) + ip(x)\gamma_5 + m]h(x) \\
&= h^\dagger(x)(D_x + m)h(x)
\end{aligned} \tag{23}$$

Here the covariance is due to the property of constant m which leads

$$h^\dagger(x)mh(x) = m \quad , \tag{24}$$

and then $\bar{\psi}(D+m)\psi$ is invariant

$$\bar{\psi}(D+m)\psi \rightarrow \bar{\psi}'(D+m)'\psi' \equiv \bar{\psi}hh^\dagger(D+m)hh^\dagger\psi = \bar{\psi}(D+m)\psi \tag{25}$$

If we change m to $\Sigma(-\partial^2)$, relation (23) is no longer valid due to differential operator dependence of Σ ,

$$h^\dagger(x)\Sigma(-\partial_x^2)h(x) = \Sigma[-h^\dagger(x)\partial_x^2 h(x)] = \Sigma\left[-\left(\partial_\mu + h^\dagger(x)[\partial_\mu h(x)]\right)^2\right] \neq \Sigma(-\partial_x^2) . \quad (26)$$

So, naive generalization to DCSB Lagrangian $\bar{\psi}[D + \Sigma(-\partial^2)]\psi$ is not invariant on $U_V(N_f)$ symmetry. To implement this local symmetry (22), instead of considering $\Sigma(-\partial^2)$, we need to consider $\Sigma(-\bar{\nabla}^2)$ in which

$$\bar{\nabla}^\mu \equiv \partial^\mu - iv^\mu(x) , \quad (27)$$

the bar over ∇_μ is to specify the difference of present derivative with that introduced in (1). Use (22), we find $\bar{\nabla}_\mu$ transform as

$$\bar{\nabla}_x^\mu \rightarrow \bar{\nabla}_x^{\mu'} \equiv \partial_x^\mu - iv^{\mu'}(x) = h^\dagger(x)\bar{\nabla}_x^\mu h(x) . \quad (28)$$

Then

$$\Sigma(-\bar{\nabla}_x^2) \rightarrow \Sigma(-\bar{\nabla}_x^{2'}) = \Sigma[-h^\dagger(x)\bar{\nabla}_x^2 h(x)] = h^\dagger(x)\Sigma(-\bar{\nabla}_x^2)h(x) \quad (29)$$

and

$$[D_x + \Sigma(-\bar{\nabla}_x^2)] \rightarrow [D'_x + \Sigma(-\bar{\nabla}_x^{2'})] = h^\dagger(x)[D_x + \Sigma(-\bar{\nabla}_x^2)]h(x) . \quad (30)$$

So, $\bar{\psi}[D + \Sigma(-\bar{\nabla}^2)]\psi$ is invariant under transformation (22) and can be treated as correct generalization of $\bar{\psi}(D + m)\psi$. Note, in principle, one can add in $\bar{\nabla}^\mu$ other terms which are covariant under transformation (22), such as $a_\mu \gamma_5$ multiply by a constant. For these kind terms, symmetry itself is not enough to fix them completely, therefore they can be treated as some more interactions which are beyond our choice of $\bar{\nabla}^\mu$ and $\Sigma(-\bar{\nabla}^2)$. Our choice of $\Sigma(-\bar{\nabla}^2)$ is the minimal generalization required by symmetry to incorporating in fermion self energy into the theory.

The generalized real part of fermion determinant now is

$$\begin{aligned} \text{Re ln Det}[D + \Sigma(-\bar{\nabla}^2)] &= \frac{1}{2} \text{Tr ln} \left[[D^\dagger + \Sigma(-\bar{\nabla}^2)][D + \Sigma(-\bar{\nabla}^2)] \right] \\ &= -\frac{1}{2} \lim_{\Lambda \rightarrow \infty} \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{d\tau}{\tau} \text{Tre}^{-\tau[\bar{E} - \nabla^2 + \Sigma^2(-\bar{\nabla}^2) + Jg(\bar{\nabla}^2) + \tilde{g}(\bar{\nabla}^2)K - \not{d} \Sigma(-\bar{\nabla}^2)]} \end{aligned} \quad (31)$$

where

$$\bar{E} - \nabla^2 + \Sigma^2(-\bar{\nabla}^2) + Jg(\bar{\nabla}^2) + \tilde{g}(\bar{\nabla}^2)K - \not{d} \Sigma(-\bar{\nabla}^2) = [D^\dagger + \Sigma(-\bar{\nabla}^2)][D + \Sigma(-\bar{\nabla}^2)] \quad (32)$$

and

$$\begin{aligned}
[\not{d} \Sigma(-\bar{\nabla}^2)] &\equiv \gamma^\mu [d_\mu \Sigma(-\bar{\nabla}^2)] = \gamma^\mu \left(\partial_\mu \Sigma(-\bar{\nabla}^2) - i[v_\mu, \Sigma(-\bar{\nabla}^2)] \right) \\
\bar{E} &\equiv \frac{i}{4} [\gamma^\mu, \gamma^\nu] \mathcal{R}_{\mu\nu} + \gamma_\mu d^\mu (s - ip\gamma_5) + i\gamma^\mu [a_\mu \gamma_5 (s - ip\gamma_5) + (s - ip\gamma_5) a_\mu \gamma_5] + s^2 + p^2 - [s, p] i\gamma_5 \\
g(x) = \tilde{g}(x) &\equiv \Sigma(-x) \quad J = -i\not{d} \gamma_5 - s - ip\gamma_5 \quad K = -i\not{d} \gamma_5 - s + ip\gamma_5 .
\end{aligned}$$

The generalized coincidence limit of fermion propagator is

$$\begin{aligned}
&[D + \Sigma(-\bar{\nabla}^2)]^{-1}(x, x) \\
&= \left[\left([D^\dagger + \Sigma(-\bar{\nabla}^2)] [D + \Sigma(-\bar{\nabla}^2)] \right)^{-1} [D^\dagger + \Sigma(-\bar{\nabla}^2)] \right](x, x) \\
&= \lim_{\Lambda \rightarrow \infty} \int_{\frac{1}{\Lambda^2}}^{\infty} d\tau \langle x | e^{-\tau [\bar{E} - \nabla^2 + \Sigma^2(-\bar{\nabla}^2) + Jg(\bar{\nabla}^2) + \tilde{g}(\bar{\nabla}^2)K - \not{d} \Sigma(-\bar{\nabla}^2)]} [-\not{\nabla} - s(x) - ip(x)\gamma_5 \\
&\quad - 2i\not{d}(x)\gamma_5 + \Sigma(-\bar{\nabla}^2)] | x \rangle
\end{aligned} \tag{33}$$

For safety of further calculation, we limit ultraviolet behavior of $\Sigma(k^2)$ satisfy constraint

$$\frac{\Sigma^2(k^2)}{k^2} \xrightarrow{k^2 \rightarrow \infty} 0 \tag{34}$$

For fermion determinant and coincidence limit of propagator, (31) and (33) tell us that the key now is to calculate matrix element of

$$\begin{aligned}
&\langle x | e^{-\tau [\bar{E} - \nabla^2 + \Sigma^2(-\bar{\nabla}^2) + Jg(\bar{\nabla}^2) + \tilde{g}(\bar{\nabla}^2)K - \not{d} \Sigma(-\bar{\nabla}^2)]} | x \rangle \\
&\text{and}
\end{aligned} \tag{35}$$

$$\langle x | e^{-\tau [\bar{E} - \nabla^2 + \Sigma^2(-\bar{\nabla}^2) + Jg(\bar{\nabla}^2) + \tilde{g}(\bar{\nabla}^2)K - \not{d} \Sigma(-\bar{\nabla}^2)]} [-\not{\nabla} - s(x) - ip(x)\gamma_5 - 2i\not{d}(x)\gamma_5 + \Sigma(-\bar{\nabla}^2)] | x \rangle$$

which are much more complex than (11) and (20). Since now the operator on the exponential is beyond original second- order elliptic partial differential operator, Σ in principle include arbitrary high order of differential operator and except the constraint (34), detail function Σ on the differential operator is still unspecified.

In original Schwinger proper time formulation, there are two methods to calculate matrix elements (11) and (20). One is based on recursion formula and the other is momentum space calculation. Recursion formula can be obtained only if we can obtain the matrix element with vanishing external fields which now is impossible, since unspecified detail momentum dependence of $\Sigma(k^2)$ prevent us to obtain it. In this work, we take second method to calculate the matrix element in momentum space,

$$\begin{aligned}
&\langle x | e^{-\tau [\bar{E} - \nabla^2 + \Sigma^2(-\bar{\nabla}^2) + Jg(\bar{\nabla}^2) + \tilde{g}(\bar{\nabla}^2)K - \not{d} \Sigma(-\bar{\nabla}^2)]} | x \rangle \\
&= \int \frac{d^4 k}{(2\pi)^4} \exp \left\{ -\tau \left[\bar{E}(x) - \nabla_x^2 - 2ik \cdot \nabla_x + k^2 + \Sigma^2(-\bar{\nabla}^2 - 2ik \cdot \bar{\nabla}_x + k^2) \right. \right. \\
&\quad \left. \left. + Jg(\bar{\nabla}_x^2 + 2ik \cdot \bar{\nabla}_x - k^2) + \tilde{g}(\bar{\nabla}_x^2 + 2ik \cdot \bar{\nabla}_x - k^2)K - \not{d} \Sigma(-\bar{\nabla}_x^2 - 2ik \cdot \bar{\nabla}_x + k^2) \right] \right\} . \tag{36}
\end{aligned}$$

Assigning \overline{E}, J, K to be order of p , we can take low energy expansion for (36). The result is given in appendix B. Similarly

$$\begin{aligned}
& \langle x | e^{-\tau[\overline{E} - \nabla^2 + \Sigma^2(-\overline{\nabla}^2) + Jg(\overline{\nabla}^2) + \tilde{g}(\overline{\nabla}^2)K - \not{d}\Sigma(-\overline{\nabla}^2)]} [-\not{\nabla} - s(x) - ip(x)\gamma_5 - 2i\not{d}(x)\gamma_5 + \Sigma(-\overline{\nabla}^2)] | x \rangle \\
&= \int \frac{d^4k}{(2\pi)^4} \exp \left\{ -\tau \left[\overline{E}(x) - \nabla_x^2 - 2ik \cdot \nabla_x + k^2 + \Sigma^2(-\overline{\nabla}^2 - 2ik \cdot \overline{\nabla}_x + k^2) \right. \right. \\
&\quad \left. \left. + Jg(\overline{\nabla}_x^2 + 2ik \cdot \overline{\nabla}_x - k^2) + \tilde{g}(\overline{\nabla}_x^2 + 2ik \cdot \overline{\nabla}_x - k^2)K - \not{d}\Sigma(-\overline{\nabla}_x^2 - 2ik \cdot \overline{\nabla}_x + k^2) \right] \right\} \\
&\quad \times [-\not{\nabla}_x - i\not{k} - s(x) - ip(x)\gamma_5 - 2i\not{d}(x)\gamma_5 + \Sigma(-\overline{\nabla}_x^2 - 2ik \cdot \overline{\nabla}_x + k^2)] | x \rangle . \quad (37)
\end{aligned}$$

We donot write down the result of expansion for this matrix element explicitly. Instead we give the final result in terms of external fields s, p, v_μ, a_μ for coincidence limit of fermion propagator and discuss it in section V.

We must note that the difficulty of present calculation is that the momentum integration now can not be finished due to unknown behavior of $\Sigma(k^2)$. While the chiral covariance of the result rely on the achievement of momentum integration in original Schwinger proper time formulation. We need to find a way to keep the covariance of chiral symmetry before finishing the momentum integration. Fortunately, we found that those non-covariant terms are all reduced to total divergence terms at momentum space which with constraint (34) vanish. So with invention of these total divergence terms, we can still obtain a chiral covariant result as that in conventional Schwinger proper time method even before we analytically finish the momentum integration.

IV. FERMION DETERMINANT WITH MOMENTUM DEPENDENT FERMION SELF ENERGY

We can parametrize the result fermion determinant as

$$\begin{aligned}
& \text{Re ln Det}[D + \Sigma(-\overline{\nabla}^2)] \\
&= \int d^4x \text{tr}_f \left[\mathcal{C}_0 s + \mathcal{C}_1 a^2 + \mathcal{C}_2 [d_\mu a^\mu]^2 + \mathcal{C}_3 (d^\mu a^\nu - d^\nu a^\mu)(d_\mu a_\nu - d_\nu a_\mu) + \mathcal{C}_4 a^4 \right. \\
&\quad \left. + \mathcal{C}_5 a^\mu a^\nu a_\mu a_\nu + \mathcal{C}_6 s^2 + \mathcal{C}_7 p^2 + \mathcal{C}_8 s a^2 + \mathcal{C}_9 V^{\mu\nu} V_{\mu\nu} + \mathcal{C}_{10} V^{\mu\nu} a_\mu a_\nu + \mathcal{C}_{11} p d_\mu a^\mu \right] + O(p^6) \quad (38)
\end{aligned}$$

Substitute the definition of \overline{E}, J, K, g and \tilde{g} into result (B1) given in appendix B, we obtain coefficients \mathcal{C}_i which are related to Σ by

$$\begin{aligned}
\mathcal{C}_0 &= -4 \int d\tilde{k} \Sigma_k X_k \\
\mathcal{C}_1 &= 2 \int d\tilde{k} \left[(-2\Sigma_k^2 + k^2 \Sigma_k \Sigma'_k) X_k^2 + (-2\Sigma_k^2 + k^2 \Sigma_k \Sigma'_k) \frac{X_k}{\Lambda^2} \right]
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_2 &= 2 \int d\tilde{k} \left[2A_k X_k^3 + 2A_k \frac{X_k^2}{\Lambda^2} + A_k \frac{X_k}{\Lambda^4} + \frac{k^2}{2} \Sigma_k' \frac{X_k}{\Lambda^2} + \frac{k^2}{2} \Sigma_k'^2 X_k^2 \right] \\
\mathcal{C}_3 &= \int d\tilde{k} \left[2B_k X_k^3 + 2B_k \frac{X_k^2}{\Lambda^2} + B_k \frac{X_k}{\Lambda^4} + \frac{k^2}{2} \Sigma_k' \frac{X_k}{\Lambda^2} + \frac{k^2}{2} \Sigma_k'^2 X_k^2 \right] \\
\mathcal{C}_4 &= -2 \int d\tilde{k} \left[\left(\frac{4\Sigma_k^4}{3} + \frac{2k^2 \Sigma_k^2}{3} + \frac{k^4}{18} \right) (6X_k^4 + \frac{6X_k^3}{\Lambda^2} + \frac{3X_k^2}{\Lambda^4} + \frac{X_k}{\Lambda^6}) - (4\Sigma_k^2 + \frac{k^2}{2}) (2X_k^3 \right. \\
&\quad \left. + \frac{2X_k^2}{\Lambda^2} + \frac{X_k}{\Lambda^4}) + \frac{X_k}{\Lambda^2} + X_k^2 \right] \\
\mathcal{C}_5 &= - \int d\tilde{k} \left[\left(\frac{-4\Sigma_k^4}{3} - \frac{2k^2 \Sigma_k^2}{3} + \frac{k^4}{18} \right) (6X_k^4 + \frac{6X_k^3}{\Lambda^2} + \frac{3X_k^2}{\Lambda^4} + \frac{X_k}{\Lambda^6}) + 4\Sigma_k^2 (2X_k^3 + \frac{2X_k^2}{\Lambda^2} \right. \\
&\quad \left. + \frac{X_k}{\Lambda^4}) - \frac{X_k}{\Lambda^2} - X_k^2 \right] \\
\mathcal{C}_6 &= -2 \int d\tilde{k} \left[(3\Sigma_k^2 - 2k^2 \Sigma_k \Sigma_k') X_k^2 + [2\Sigma_k^2 - k^2(1 + 2\Sigma_k \Sigma_k')] \frac{X_k}{\Lambda^2} \right] \\
\mathcal{C}_7 &= -2 \int d\tilde{k} \left[(\Sigma_k^2 - 2k^2 \Sigma_k \Sigma_k') X_k^2 - k^2(1 + 2\Sigma_k \Sigma_k') \frac{X_k}{\Lambda^2} \right] \\
\mathcal{C}_8 &= -4 \int d\tilde{k} \left[(4\Sigma_k^3 + k^2 \Sigma_k) X_k^3 + (4\Sigma_k^3 + k^2 \Sigma_k) \frac{X_k^2}{\Lambda^2} + (2\Sigma_k^3 + \frac{1}{2} k^2 \Sigma_k) \frac{X_k}{\Lambda^4} - 3\Sigma_k \frac{X_k}{\Lambda^2} \right. \\
&\quad \left. - 3\Sigma_k X_k^2 \right] \\
\mathcal{C}_9 &= \int d\tilde{k} \left[\left(\frac{1}{3} k^2 \Sigma_k' \Sigma_k'' - \frac{1}{3} k^2 \Sigma_k \Sigma_k'' \right) X_k + (-C_k + D_k) \frac{X_k}{\Lambda^2} - (C_k - D_k) X_k^2 + 2E_k X_k^3 \right. \\
&\quad \left. + 2E_k \frac{X_k^2}{\Lambda^2} + E_k \frac{X_k}{\Lambda^4} \right] \\
\mathcal{C}_{10} &= 4i \int d\tilde{k} \left[2F_k X_k^3 + 2F_k \frac{X_k^2}{\Lambda^2} + F_k \frac{X_k}{\Lambda^4} + \frac{k^2}{2} \Sigma_k' \frac{X_k}{\Lambda^2} + \frac{k^2}{2} \Sigma_k'^2 X_k^2 \right] \\
\mathcal{C}_{11} &= 4 \int d\tilde{k} \left[(\Sigma_k - \frac{1}{2} k^2 \Sigma_k') \frac{X_k}{\Lambda^2} + (\Sigma_k - \frac{1}{2} k^2 \Sigma_k') X_k^2 \right] \tag{39}
\end{aligned}$$

where

$$\int d\tilde{k} \equiv \int \frac{d^4 k}{(2\pi)^4} e^{-\frac{k^2 + \Sigma^2(k^2)}{\Lambda^2}} \tag{40}$$

$$\Sigma_k \equiv \Sigma(k^2) \quad X_k \equiv \frac{1}{k^2 + \Sigma^2(k^2)} \tag{41}$$

and $A_k, B_k, C_k, D_k, E_k, F_k$ depending on $\Sigma(k^2)$ are given in appendix A. We see that asymptotic behavior of $\Sigma(k^2)$ (34) insure factor $e^{-\frac{k^2 + \Sigma^2(k^2)}{\Lambda^2}}$ appeared in integration measure (40) is an ultraviolet damping factor which will keep our momentum integration convergent.

(38) and (39) are our final result for the real part of fermion determinant with presence of dynamical quark self energy. The result in this paper is only up to order of p^4 , one can easily generalize the calculation to higher orders of the momentum expansion. As a self check of theory, take $\Sigma(k^2)$ be constant m , in the limit of $\Lambda^2 \rightarrow \infty$, the momentum integration in (39) can be finished, the result gives

$$\begin{aligned}
\mathcal{C}_0 &\xrightarrow{\Sigma=m} -\frac{N_c}{4\pi^2}m[\Lambda^2 + m^2(\ln\frac{m^2}{\Lambda^2} + \gamma - 1)] \\
\mathcal{C}_1 &\xrightarrow{\Sigma=m} \frac{N_c}{4\pi^2}m^2(\ln\frac{m^2}{\Lambda^2} + \gamma) \\
\mathcal{C}_2 &\xrightarrow{\Sigma=m} \frac{N_c}{24\pi^2} \\
\mathcal{C}_3 &\xrightarrow{\Sigma=m} \frac{N_c}{48\pi^2}(\ln\frac{m^2}{\Lambda^2} + \gamma + 1) \\
\mathcal{C}_4 &\xrightarrow{\Sigma=m} \frac{N_c}{24\pi^2}(\ln\frac{m^2}{\Lambda^2} + \gamma + 4) \\
\mathcal{C}_5 &\xrightarrow{\Sigma=m} -\frac{N_c}{24\pi^2}(\ln\frac{m^2}{\Lambda^2} + \gamma + 2) \\
\mathcal{C}_6 &\xrightarrow{\Sigma=m} \frac{N_c}{8\pi^2}\left[\Lambda^2 + m^2(3\ln\frac{m^2}{\Lambda^2} + 3\gamma - 1)\right] \\
\mathcal{C}_7 &\xrightarrow{\Sigma=m} \frac{N_c}{8\pi^2}\left[\Lambda^2 + m^2(\ln\frac{m^2}{\Lambda^2} + \gamma - 1)\right] \\
\mathcal{C}_8 &\xrightarrow{\Sigma=m} -\frac{N_c}{2\pi^2}m(\ln\frac{m^2}{\Lambda^2} + \gamma + 1) \\
\mathcal{C}_9 &\xrightarrow{\Sigma=m} \frac{N_c}{48\pi^2}(\ln\frac{m^2}{\Lambda^2} + \gamma) \\
\mathcal{C}_{10} &\xrightarrow{\Sigma=m} -\frac{iN_c}{12\pi^2}(\ln\frac{m^2}{\Lambda^2} + \gamma + 2) \\
\mathcal{C}_{11} &\xrightarrow{\Sigma=m} -\frac{N_c}{4\pi^2}m(\ln\frac{m^2}{\Lambda^2} + \gamma) \quad .
\end{aligned} \tag{42}$$

Substitute them back into (38), we reproduce original result (12). So our generalized formulation can easily recover the conventional Schwinger proper time result.

V. COINCIDENCE LIMIT OF FERMION PROPAGATOR WITH MOMENTUM DEPENDENT FERMION SELF ENERGY

The computation of (33) shows that the scalar, pseudoscalar and axial vector parts of coincidence limit of fermion propagator are

$$\begin{aligned}
&\frac{1}{2}(1)^{\xi\zeta}\left[[D + \Sigma(-\overline{\nabla}^2)]^{-1,(b\zeta)(a\xi)}(x, x) + [D^\dagger + \Sigma(-\overline{\nabla}^{\dagger 2})]^{-1,(b\zeta)(a\xi)}(x, x)\right] \\
&= -\left[\mathcal{C}_0 + 2\mathcal{C}_6s + \mathcal{C}_8a^2\right]^{ba} + O(p^4) \quad ,
\end{aligned} \tag{43}$$

$$\begin{aligned}
&\frac{1}{2}(\gamma_5)^{\xi\zeta}\left[[D + \Sigma(-\overline{\nabla}^2)]^{-1,(b\zeta)(a\xi)}(x, x) - [D^\dagger + \Sigma(-\overline{\nabla}^{\dagger 2})]^{-1,(b\zeta)(a\xi)}(x, x)\right] \\
&= -i\left[2\mathcal{C}_7p + \mathcal{C}_{11}d^\mu a_\mu\right]^{ba} + O(p^4) \quad ,
\end{aligned} \tag{44}$$

$$\frac{1}{2}(\gamma_5\gamma_\mu)^{\xi\zeta}\left[[D + \Sigma(-\overline{\nabla}^2)]^{-1,(b\zeta)(a\xi)}(x, x) + [D^\dagger + \Sigma(-\overline{\nabla}^{\dagger 2})]^{-1,(b\zeta)(a\xi)}(x, x)\right]$$

$$\begin{aligned}
&= -i \left[2\mathcal{C}_1 a^\mu - 2\mathcal{C}_2 [d^\mu d^\nu a_\nu] + 2\mathcal{C}_3 d^\nu (d^\mu a_\nu - d_\nu a^\mu) + 2\mathcal{C}_4 (a^2 a^\mu + a^\mu a^2) + 4\mathcal{C}_5 a_\nu a^\mu a^\nu \right. \\
&\quad \left. + \mathcal{C}_8 (s a^\mu + a^\mu s) + \mathcal{C}_{10} (a_\nu V^{\mu\nu} - V^{\mu\nu} a_\nu) - \mathcal{C}_{11} d^\mu p \right]^{ba} + O(p^5) , \tag{45}
\end{aligned}$$

where the coefficients \mathcal{C}_i are given in last section. Note the computation is done for fermion determinant and propagator independently, and for scalar, pseudoscalar and axial vector parts, we recover the result of which we directly performing functional differential with scalar, pseudo scalar and axial vector fields for determinant. Unfortunately, this property no longer valid for vector part of propagator. We parametrize the corresponding coincidence limit of fermion propagator as

$$\begin{aligned}
&2(\gamma_\mu)^{\xi\zeta} \left[[D + \Sigma(-\overline{\nabla}^2)]^{-1, (b\zeta)(a\xi)}(x, x) - [D^\dagger + \Sigma(-\overline{\nabla}^{\dagger 2})]^{-1, (b\zeta)(a\xi)}(x, x) \right] \\
&= i \left[2i\overline{\mathcal{C}}_2 [d_\nu a^\nu, a^\mu] + 4i\overline{\mathcal{C}}_3 [d^\mu a^\nu - d^\nu a^\mu, a_\nu] + 4\overline{\mathcal{C}}_9 d^\nu V^{\mu\nu} + \overline{\mathcal{C}}_{10} d_\nu [a^\mu, a^\nu] + i\overline{\mathcal{C}}_{11} [p, a^\mu] \right]^{ba} \\
&\quad + O(p^5) , \tag{46}
\end{aligned}$$

The coefficients $\overline{\mathcal{C}}_i$ with overline on them are to specify the difference with their original coefficient \mathcal{C}_i from functional differential with vector fields. We list down their values in appendix C

Now we discuss the reason to cause this difference. With momentum dependent fermion self energy, the coincidence limit of fermion propagator can not be obtained directly as that in hard mass case from the functional differential of external fields for fermion determinant. Since the formula corresponding to (14) now is changed to

$$\begin{aligned}
&\frac{\delta \text{Tr} \ln [D + \Sigma(-\overline{\nabla}^2)]}{\delta J^{\sigma\rho}(x)} \\
&= \int d^4 y d^4 z [D + \Sigma(-\overline{\nabla}^2)]^{-1, \rho'\sigma'}(y, z) \frac{\delta [D + \Sigma(-\overline{\nabla}^2)]^{\sigma'\rho'}(z, y)}{\delta J^{\sigma\rho}(x)} \\
&= [D + \Sigma(-\overline{\nabla}^2)]^{-1, \rho\sigma}(x, x) + \int d^4 y d^4 z [D + \Sigma(-\overline{\nabla}^2)]^{-1, \rho'\sigma'}(y, z) \frac{\delta [\Sigma(-\overline{\nabla}^2)]^{\sigma'\rho'}(z, y)}{\delta J^{\sigma\rho}(x)} \tag{47}
\end{aligned}$$

and its hermitian conjugate formula. The second term in (47) is a new term which rely on the momentum dependent $\Sigma(k^2)$ and vanishes when self energy is independent of external fields. Because of this extra term, functional differential of external fields for fermion determinant no longer give propagator. Note that $\Sigma(-\overline{\nabla}^2)$ in the second term is independent of scalar, pseudo scalar and axial vector fields and the corresponding differential with scalar, pseudo scalar and axial vector fields vanish, so this extra term donot contribute to scalar, pseudo scalar and axial vector parts of coincidence limit of fermion propagator. This explains why

direct calculation of scalar, pseudo scalar and axial vector parts of coincidence limit of propagator can recover result of functional differential for determinant.

Parametrize the second term as

$$\begin{aligned}
& \frac{1}{2}(\gamma_\mu)^{\xi\zeta} \int d^4y d^4z \left[[D + \Sigma(-\bar{\nabla}^2)]^{-1, \rho'\sigma'}(y, z) \frac{\delta[\Sigma(-\bar{\nabla}^2)]^{\sigma'\rho'}(z, y)}{\delta J^{(a\xi)(b\zeta)}(x)} \right. \\
& \left. - [D^\dagger + \Sigma(-\bar{\nabla}^{\dagger 2})]^{-1, \rho'\sigma'}(y, z) \frac{\delta[\Sigma(-\bar{\nabla}^{\dagger 2})]^{\sigma'\rho'}(z, y)}{\delta J^{\dagger(a\xi)(b\zeta)}(x)} \right] \\
& = i \left[2i\tilde{\mathcal{C}}_2[d_\nu a^\nu, a^\mu] + 4i\tilde{\mathcal{C}}_3[d^\mu a^\nu - d^\nu a^\mu, a_\nu] + 4\tilde{\mathcal{C}}_9 d^\nu V^{\mu\nu} + \tilde{\mathcal{C}}_{10} d_\nu[a^\mu, a^\nu] + i\tilde{\mathcal{C}}_{11}[p, a^\mu] \right]^{ba} \\
& \quad + O(p^5) \ , \tag{48}
\end{aligned}$$

We can use our generalized Schwinger proper time method directly calculate these coefficients.

$$\begin{aligned}
& \text{Tr} \left[[D + \Sigma(-\bar{\nabla}^2)]^{-1} \frac{\delta[D + \Sigma(-\bar{\nabla}^2)]}{\delta J^{\sigma\rho}(x)} \right] \\
& = \text{Tr} \left[\left([D^\dagger + \Sigma(-\bar{\nabla}^2)][D + \Sigma(-\bar{\nabla}^2)] \right)^{-1} [D^\dagger + \Sigma(-\bar{\nabla}^2)] \frac{\delta[D + \Sigma(-\bar{\nabla}^2)]}{\delta J^{\sigma\rho}(x)} \right] \\
& = \lim_{\Lambda \rightarrow \infty} \int_{\frac{1}{\Lambda^2}}^\infty d\tau \int d^4y \text{tr} \langle y | e^{-\tau[\bar{E} - \nabla^2 + \Sigma^2(-\bar{\nabla}^2) + Jg(\bar{\nabla}^2) + \bar{g}(\bar{\nabla}^2)K - \not{d}\Sigma(-\bar{\nabla}^2)]} [-\not{\nabla} - s(x) - ip(x)\gamma_5 \\
& \quad - 2i\not{d}(x)\gamma_5 + \Sigma(-\bar{\nabla}^2)] \frac{\delta[D + \Sigma(-\bar{\nabla}^2)]}{\delta J^{\sigma\rho}(x)} | y \rangle \ . \tag{49}
\end{aligned}$$

Combine above result and its hermitian conjugate together, we can calculate coefficients $\tilde{\mathcal{C}}$ in (48). We list down the result coefficients $\tilde{\mathcal{C}}$ in appendix C. One can check that summation over $\bar{\mathcal{C}}$ and $\tilde{\mathcal{C}}$ coefficients together recover original \mathcal{C} coefficients:

$$\mathcal{C}_i = \bar{\mathcal{C}}_i + \tilde{\mathcal{C}}_i \quad i = 2, 3, 9, 10, 11 \tag{50}$$

VI. SUMMARY AND DISCUSSION

In this paper, we have generalized conventional Schwinger proper time method for standard Dirac operator to incorporate dynamical chiral symmetry breaking. The physical output of this generalization is the real part of logarithm of fermion determinant and coincidence limit of fermion propagator in presence of momentum dependent fermion self energy $\Sigma(k^2)$. The mathematical progress of this generalization is that the operator on the exponential in the key matrix element of Schwinger proper time formulation is now generalized from original second-order elliptic differential operator $E - \nabla^2$ (see (11)) to arbitrary high

order differential operator $\overline{E} - \nabla^2 + \Sigma^2(-\overline{\nabla}^2) + Jg(\overline{\nabla}^2) + \tilde{g}(\overline{\nabla}^2)K - \not{d} \Sigma(-\overline{\nabla}^2)$ (see (35)). This high order differential operator dependence is represented by an unspecified function $\Sigma(-\overline{\nabla}^2)$ which physically is fermion self energy and characterize the dynamical chiral symmetry breaking of the system. The generalized formulation automatically keep the local symmetry $U_V(N_f)$ given by (22) which is residual symmetry after explicitly breaking of local chiral symmetry $U_L(N_f) \otimes U_R(N_f)$ by constant fermion mass.

One can further generalize this local symmetry $U_V(N_f)$ invariance of the formulation to the invariance of full chiral symmetry $U_L(N_f) \otimes U_R(N_f)$. The price is that the symmetry must be realized nonlinearly, i.e., we need to introduce into theory a local field $\Omega(x)$ which transform nonlinearly under local chiral symmetry (3) as

$$\Omega(x) \rightarrow \Omega'(x) = h^\dagger(x)\Omega(x)V_L^\dagger(x) = V_R(x)\Omega(x)h(x) \quad (51)$$

and rotated external fields

$$\begin{aligned} J_\Omega(x) &= [\Omega(x)P_R + \Omega^\dagger(x)P_L] [J(x) + i\not{\partial}] [\Omega(x)P_R + \Omega^\dagger(x)P_L] \\ &\equiv -s_\Omega(x) + ip_\Omega(x)\gamma_5 + \not{p}_\Omega(x) + \not{d}_\Omega(x)\gamma_5 \end{aligned} \quad (52)$$

in which $h(x)$ is a transformation matrix for induced hidden local symmetry $U_V(N_f)$. Replacing the external fields in $D + \Sigma(-\overline{\nabla}^2)$ in (31) and (33) with $D_\Omega + \Sigma(-\overline{\nabla}_\Omega^2)$, i.e.

$$D + \Sigma(-\overline{\nabla}^2) \rightarrow D_\Omega + \Sigma(-\overline{\nabla}_\Omega^2) \quad (53)$$

with

$$\begin{aligned} D_\Omega &\equiv \not{\nabla}_\Omega - s_\Omega + ip_\Omega\gamma_5 \\ \nabla_\Omega^\mu &\equiv \partial^\mu - iv_\Omega^\mu - ia_\Omega^\mu\gamma_5 \\ \overline{\nabla}_\Omega^\mu &\equiv \partial^\mu - iv_\Omega^\mu(x) \ , \end{aligned} \quad (54)$$

One can easily check that under local chiral symmetry $U_L(N_f) \otimes U_R(N_f)$ transformation (3) and (51),

$$J_\Omega(x) \rightarrow J'_\Omega(x) = h^\dagger(x)[J_\Omega(x) + i\not{\partial}_x]h(x) \quad (55)$$

$$D_\Omega + \Sigma(-\overline{\nabla}_\Omega^2) \rightarrow D'_\Omega + \Sigma(-\overline{\nabla}_\Omega'^2) = h^\dagger(x)[D_\Omega + \Sigma(-\overline{\nabla}_\Omega^2)]h(x) \quad (56)$$

i.e., they are covariant quantities. In terms of rotated quantities, local chiral symmetry $U_L(N_f) \otimes U_R(N_f)$ is nonlinearly realized through hidden symmetry $U_V(N_f)$ by $h(x)$. Once our formulation keep this local hidden symmetry $U_V(N_f)$, it automatically keep original

local chiral symmetry $U_L(N_f) \otimes U_R(N_f)$. Since our generalized formulation do keep the local symmetry $U_V(N_f)$, it then keep local chiral symmetry $U_L(N_f) \otimes U_R(N_f)$ in terms of Ω field.

In this paper, we do not calculate the imaginary part of fermion determinant and fermion propagator are only limited to its coincidence limit, its general nonlocal part is not computed. These are all under investigation and we will present results in separated papers.

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APPENDIX A: COEFFICIENTS DEFINITIONS

$$\begin{aligned}
A_k &= \frac{2}{3}k^2\Sigma_k\Sigma'_k(-1-2\Sigma_k\Sigma'_k) - \frac{1}{3}\Sigma_k^2(-1-2\Sigma_k\Sigma'_k) + \frac{1}{3}k^2\Sigma_k^2(\Sigma_k'^2 + \Sigma_k\Sigma_k'') \\
&\quad + \frac{1}{6}k^4(\Sigma_k'^2 + \Sigma_k\Sigma_k'') \\
B_k &= \frac{2}{3}k^2\Sigma_k\Sigma'_k(-1-2\Sigma_k\Sigma'_k) - \frac{1}{3}\Sigma_k^2(-1-2\Sigma_k\Sigma'_k) + \frac{1}{3}k^2\Sigma_k^2(\Sigma_k'^2 + \Sigma_k\Sigma_k'') \\
&\quad + \frac{1}{18}k^4(\Sigma_k'^2 + \Sigma_k\Sigma_k'') + \frac{1}{6}k^2(-1-2\Sigma_k\Sigma'_k) \\
C_k &= \frac{1}{3} - \frac{1}{3}\Sigma_k\Sigma'_k + \frac{1}{2}k^2\Sigma_k'^2 \\
D_k &= -\frac{1}{2}k^2\Sigma_k'^2 + \frac{1}{6}k^2\Sigma_k\Sigma_k''(-1-2\Sigma_k\Sigma'_k) + \frac{2}{9}k^4\Sigma_k'\Sigma_k''(1+2\Sigma_k\Sigma'_k) - \frac{2}{9}k^4\Sigma_k'^2(-\Sigma_k'^2 - \Sigma_k\Sigma_k'') \\
&\quad + \frac{1}{3}k^2\Sigma_k\Sigma'_k(-\Sigma_k'^2 - \Sigma_k\Sigma_k'') \\
E_k &= \frac{1}{6}k^2\Sigma_k\Sigma'_k(-1-2\Sigma_k\Sigma'_k)^2 - \frac{1}{9}k^4\Sigma_k'^2(1+2\Sigma_k\Sigma'_k)^2 \\
F_k &= \frac{4}{3}k^2\Sigma_k\Sigma'_k - \frac{4}{3}k^2(\Sigma_k\Sigma'_k)^2 - \frac{2}{3}\Sigma_k^2 + \frac{2}{3}\Sigma_k^3\Sigma'_k + \frac{1}{3}k^2\Sigma_k^2(\Sigma_k'^2 + \Sigma_k\Sigma_k'') + \frac{1}{9}k^4(\Sigma_k'^2 + \Sigma_k\Sigma_k'') \\
&\quad + \frac{1}{3}k^2(-1-2\Sigma_k\Sigma'_k) + \frac{1}{2}k^2
\end{aligned}$$

APPENDIX B: GENERALIZED SEELY-DEWITT EXPANSION

The detail calculation gives:

$$\begin{aligned}
&\langle x|e^{-\tau[\bar{E}-\nabla^2+\Sigma^2(-\bar{\nabla}^2)+Jg(\bar{\nabla}^2)+\tilde{g}(\bar{\nabla}^2)K-\not{d}\Sigma(-\bar{\nabla}^2)]}|x\rangle \\
&= \int \frac{d^4k}{(2\pi)^4} e^{-\tau f} \left\{ Y_1 + k^2 Y_2 + k^4 Y_3 + \frac{1}{8}k^2 \left(-\frac{\tau}{3}f'' + \tau^2 f' f'' - \frac{\tau^3}{3}f'^3 \right) [\bar{\nabla}^\mu, \bar{\nabla}^\nu][\bar{\nabla}_\mu, \bar{\nabla}_\nu] \right\}
\end{aligned}$$

$$\begin{aligned}
& -i\frac{\tau^2}{36}f'''k^4\gamma_5([\bar{\nabla}^\mu, [\bar{\nabla}_\mu, \bar{\nabla}^\nu]]a_\nu + a_\nu[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, \bar{\nabla}^\nu]]) + i\frac{\tau^3}{36}f'f''k^4\gamma_5(-2[a^\nu, [\bar{\nabla}^\mu, [\bar{\nabla}_\mu, \bar{\nabla}_\nu]]) \\
& + [\bar{\nabla}^\mu, \bar{\nabla}^\nu][\bar{\nabla}_\nu, a_\mu] + [\bar{\nabla}_\nu, a_\mu][\bar{\nabla}^\mu, \bar{\nabla}^\nu] + [\bar{\nabla}_\nu, [\bar{\nabla}^\mu, \bar{\nabla}^\nu]a_\mu] + [\bar{\nabla}_\nu, a_\mu[\bar{\nabla}^\mu, \bar{\nabla}^\nu]]) \\
& -i\frac{\tau^4}{36}f'^3\gamma_5([\bar{\nabla}_\nu, a_\mu][\bar{\nabla}^\mu, \bar{\nabla}^\nu] + [\bar{\nabla}^\mu, \bar{\nabla}^\nu][\bar{\nabla}_\nu, a_\mu]) - \frac{\tau^3}{12}f'^2k^2[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, a^2]]) \\
& -\frac{\tau^3}{36}f''k^4([\bar{\nabla}^\mu, a_\mu][\bar{\nabla}^\nu, a_\nu] + 2[\bar{\nabla}^\nu, a^\mu][\bar{\nabla}_\mu, a_\nu] - 2[\bar{\nabla}^\mu, [\bar{\nabla}^\nu, a_\mu a_\nu]] - 2a_\mu a_\nu[\bar{\nabla}^\mu, \bar{\nabla}^\nu]) \\
& +\frac{\tau^4}{36}f'^2k^4(a^\nu[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, a_\nu]] + [\bar{\nabla}^\mu, [\bar{\nabla}_\mu, a_\nu]]a^\nu + [\bar{\nabla}^\mu, a^\nu][\bar{\nabla}_\mu, a_\nu] + [\bar{\nabla}^\nu, a^\mu][\bar{\nabla}_\mu, a_\nu] \\
& -a_\mu a_\nu[\bar{\nabla}^\mu, \bar{\nabla}^\nu] - [\bar{\nabla}^\mu, \bar{\nabla}^\nu]a_\mu a_\nu) - \frac{\tau^3}{12}f'k^2\gamma_5([a^\mu, [\bar{\nabla}^\mu, a^2]] + [\bar{\nabla}^\mu, [a_\mu, a^2]]) \\
& +\frac{i\tau^4}{36}f'k^4\gamma_5([\bar{\nabla}^\mu, a^\nu][a_\nu, a_\mu] + [a_\nu, a_\mu][\bar{\nabla}^\mu, a^\nu] + [\bar{\nabla}^\mu, [a_\mu, a^2]] + [a^\mu, [\bar{\nabla}^\mu, a^2]]) \\
& -\frac{\tau^3}{12}f'^2k^2[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, F]] + \frac{\tau^2}{4}f'k^2[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, F']] + \frac{\tau^2}{4}k^2(g'^2J[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, J]] \\
& +\tilde{g}'^2[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, K]]K + g'\tilde{g}'[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, KJ]]) - \frac{\tau^3}{12}f'k^2(2F'[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, F]] \\
& +2[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, F']]F + 2[\bar{\nabla}^\mu, F'][\bar{\nabla}_\mu, F] + 2F[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, F']] + 2[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, F]]F' \\
& +2[\bar{\nabla}^\mu, F][\bar{\nabla}_\mu, F'] - [\bar{\nabla}^\mu, [\bar{\nabla}_\mu, Jg'F + F\tilde{g}'K]] - [\bar{\nabla}^\mu, [\bar{\nabla}_\mu, F]]Jg' - \tilde{g}'K[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, F]) \\
& +\frac{\tau^4}{24}f'^2k^2(F[\bar{\nabla}^\mu, [\bar{\nabla}_\mu, F]] + [\bar{\nabla}^\mu, [\bar{\nabla}_\mu, F]]F + [\bar{\nabla}^\mu, F][\bar{\nabla}_\mu, F]) \\
& +\frac{\tau^2}{4}k^2\gamma_5[g'(a_\mu[\bar{\nabla}^\mu, J] + [\bar{\nabla}^\mu, a_\mu J] - J[\bar{\nabla}^\mu, a_\mu]) + \tilde{g}'([\bar{\nabla}^\mu, a_\mu]K - [\bar{\nabla}^\mu, Ka_\mu] - [\bar{\nabla}^\mu, K]a_\mu)] \\
& -\frac{i\tau^3}{12}f'k^2\gamma_5([\bar{\nabla}^\mu, [a_\mu, F]] + [a_\mu, [\bar{\nabla}^\mu, F]]) - \frac{\tau^3}{12}k^2\gamma_5(F'[\bar{\nabla}^\mu, [a_\mu, F]] + [a_\mu, [\bar{\nabla}^\mu, F']]F \\
& +[a_\mu, F'][\bar{\nabla}^\mu, F] + F'[\bar{\nabla}^\mu, [a_\mu, F]] + [\bar{\nabla}^\mu, [a_\mu, F']]F + [\bar{\nabla}^\mu, F']a_\mu) \\
& -\frac{i\tau^3}{12}g'k^2\gamma_5[g([\bar{\nabla}^\mu, J]a_\mu F - a_\mu J[\bar{\nabla}^\mu, F] - J[\bar{\nabla}^\mu, a_\mu F] + [\bar{\nabla}^\mu, J]Fa_\mu + F[\bar{\nabla}^\mu, J]a_\mu \\
& +[\bar{\nabla}^\mu, Fa_\mu J] + a_\mu F[\bar{\nabla}^\mu, J] + [\bar{\nabla}^\mu, FJ]a_\mu) + [\tilde{g}(-a_\mu[\bar{\nabla}^\mu, KF] - [\bar{\nabla}^\mu, K]Fa_\mu - [\bar{\nabla}^\mu, Ka_\mu F] \\
& -a_\mu[\bar{\nabla}^\mu, K]F - a_\mu F[\bar{\nabla}^\mu, K] + [\bar{\nabla}^\mu, Fa_\mu]K - Fa_\mu[\bar{\nabla}^\mu, K] + [\bar{\nabla}^\mu, F]Ka_\mu)] \\
& +\frac{i\tau^4}{24}f'k^2\gamma_5(F[\bar{\nabla}^\mu, [a_\mu, F]] + [\bar{\nabla}^\mu, [a_\mu\gamma_5, F]]F + [\bar{\nabla}^\mu, F][a_\mu, F] + F[a_\mu, [\bar{\nabla}^\mu, F]] \\
& +[a_\mu, [\bar{\nabla}^\mu, F]]F + [a_\mu, F][\bar{\nabla}^\mu, F]) - \frac{\tau^2}{4}\Sigma'k^2(F'[\bar{\nabla}_\mu, [-i\mathbb{V}, \bar{\nabla}^\mu]] - [\bar{\nabla}_\mu[-i\mathbb{V}, \bar{\nabla}^\mu]]F') \\
& +i\frac{\tau^2}{2}\Sigma'k^2((\tilde{g}[\bar{\nabla}^\mu, K][\mathbb{V}, \bar{\nabla}^\mu] + g[\mathbb{V}, \bar{\nabla}^\mu])[J, \bar{\nabla}^\mu]) + \frac{\tau^2}{2}\Sigma'k^2[\mathbb{V}, \bar{\nabla}^\mu]^2 \\
& +i\frac{\tau^3}{12}f'\Sigma'k^2([\bar{\nabla}_\mu, [\mathbb{V}, \bar{\nabla}^\mu]F] + [\bar{\nabla}_\mu, F[\mathbb{V}, \bar{\nabla}^\mu]] + [[\mathbb{V}, \bar{\nabla}_\mu], [\bar{\nabla}^\mu, F]]) \\
& -\frac{\tau^2}{2}\Sigma'k^2\gamma_5([\mathbb{V}, \bar{\nabla}^\mu]a_\mu + a_\mu[\mathbb{V}, \bar{\nabla}^\mu]) + \frac{\tau^3}{6}\Sigma'\gamma_5k^2(a^\mu[\mathbb{V}, \bar{\nabla}^\mu]F + a^\mu F[\mathbb{V}, \bar{\nabla}^\mu] \\
& +[\mathbb{V}, \bar{\nabla}^\mu]Fa^\mu + [\mathbb{V}, \bar{\nabla}^\mu]a^\mu F + Fa^\mu[\mathbb{V}, \bar{\nabla}^\mu] + F[\mathbb{V}, \bar{\nabla}^\mu]a^\mu)\} \\
& +\text{total derivative terms}
\end{aligned}$$

(B1)

where the total derivative terms are

total derivative terms

$$\begin{aligned}
&= \int \frac{d^4 k}{(2\pi)^4} \frac{\partial}{\partial k^\mu} k^\mu \left\{ e^{-\tau f} \left[\frac{\tau}{72} k^2 f''' + \frac{\tau^2}{8} f'' - \frac{\tau^2}{24} k^2 f' f'' + \frac{\tau^3}{72} k^2 f'^3 + \frac{\tau^2}{8} f'^2 \right] \nabla_x^4 \right. \\
&\quad - k^2 e^{-\tau f} \left[-\frac{\tau}{72} f''' + \frac{\tau^2}{24} f' f'' - \frac{\tau^3}{72} f'^3 \right] \bar{\nabla}^\mu \bar{\nabla}^2 \bar{\nabla}_\mu \\
&\quad - k^2 e^{-\tau f} \left[-\frac{\tau}{72} f''' + \frac{\tau^2}{24} f' f'' - \frac{\tau^3}{72} f'^3 \right] \bar{\nabla}^\mu \bar{\nabla}^\nu \bar{\nabla}_\mu \bar{\nabla}_\nu - \frac{i\tau}{4} k^2 \gamma_5 e^{-\tau f} (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) \\
&\quad + \frac{i\tau^2}{8} k^2 \gamma_5 e^{-\tau f} f' [\bar{\nabla}^2 (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) + (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) \bar{\nabla}^2] \\
&\quad - \frac{i\tau^2 \gamma_5}{24} k^2 e^{-\tau f} f'' [\bar{\nabla}_\mu \bar{\nabla}^2 a^\mu + \bar{\nabla}^2 \bar{\nabla}^\mu a_\mu + a^\mu \bar{\nabla}_\mu \bar{\nabla}^2 + a_\mu \bar{\nabla}^2 \bar{\nabla}^\mu] \\
&\quad + \frac{i\tau^3 \gamma_5}{72} k^2 e^{-\tau f} f'^2 [\bar{\nabla}^2 (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) + (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) \bar{\nabla}^2 + 2 \bar{\nabla}^\mu (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) \bar{\nabla}_\mu \\
&\quad + 2 \bar{\nabla}_\mu \bar{\nabla}^2 + 2 a_\mu \bar{\nabla}^2 \bar{\nabla}^\mu] + \frac{\tau^2}{8} k^2 e^{-\tau f} f' (-k^2) [\bar{\nabla}^2 a^2 + a^2 \bar{\nabla}^2] \\
&\quad + \frac{i\tau^2 \gamma_5}{8} e^{-\tau f} [a^2 (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) + (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) a^2] - \frac{\tau^2}{8} e^{-\tau f} (\bar{\nabla}^\mu a_\mu + a^\mu \bar{\nabla}_\mu)^2 \\
&\quad - \frac{\tau^3}{72} k^2 e^{-\tau f} f' [(\bar{\nabla}^\mu a_\mu + a^\mu \bar{\nabla}_\mu)^2 + 2 \bar{\nabla}^\nu (\bar{\nabla}^\mu a_\mu + a^\mu \bar{\nabla}_\mu) a_\nu + 2 a^\nu (\bar{\nabla}^\mu a_\mu + a^\mu \bar{\nabla}_\mu) \bar{\nabla}_\nu + 2 a^2 \bar{\nabla}^2 \\
&\quad + 2 \bar{\nabla}^2 a^2 + 2 a^\mu \bar{\nabla}^2 a_\mu] - \frac{i\tau^3}{36} e^{-\tau f} [a^2 (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) + (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) a^2 + a^\nu (a^\mu \bar{\nabla}_\mu + a^\mu a_\mu) a_\nu] \\
&\quad - \frac{\tau}{4} e^{-\tau f} (g' J \bar{\nabla}^2 + \tilde{g} \bar{\nabla}^2 K) + \frac{\tau^2}{8} e^{-\tau f} f' (F \bar{\nabla}^2 + \bar{\nabla}^2 F) \\
&\quad + \frac{\tau^2}{8} e^{-\tau f} [g' (J \bar{\nabla}^2 F + F J \bar{\nabla}^2) + \tilde{g} (\bar{\nabla}^2 K F + F \bar{\nabla}^2 K)] - \frac{\tau^3}{24} e^{-\tau f} (\bar{\nabla}^2 F^2 + F^2 \bar{\nabla}^2 + F \bar{\nabla}^2 F) \\
&\quad + \frac{i\tau^2 \gamma_5}{8} e^{-\tau f} [(a^\mu \bar{\nabla}_\mu + \bar{\nabla}_\mu a^\mu) F + F (a^\mu \bar{\nabla}_\mu + \bar{\nabla}_\mu a^\mu)] \\
&\quad - \frac{i\tau^3 \gamma_5}{24} e^{-\tau f} [(a^\mu \bar{\nabla}_\mu + \bar{\nabla}_\mu a^\mu) F^2 + F^2 (a^\mu \bar{\nabla}_\mu + \bar{\nabla}_\mu a^\mu) + F (a^\mu \bar{\nabla}_\mu + \bar{\nabla}_\mu a^\mu) F] \\
&\quad \left. + i \frac{\tau}{4} e^{-\tau f} \Sigma' [\nabla, \bar{\nabla}^2] - i \frac{\tau^2}{8} e^{-\tau f} \Sigma' ([\nabla, \bar{\nabla}^2] F + F [\nabla, \bar{\nabla}^2]) \right\} \tag{B2}
\end{aligned}$$

and

$$\begin{aligned}
f &\equiv k^2 + \Sigma^2(k^2) & f' &\equiv -\frac{\partial f}{\partial k^2} & f'' &\equiv \frac{\partial^2 f}{\partial (k^2)^2} & f''' &\equiv -\frac{\partial^3 f}{\partial (k^2)^3} \\
F &= \bar{E}(x) + J(x)g(-k^2) + \tilde{g}(-k^2)K(x) \\
F' &= J(x)g'(-k^2) + \tilde{g}'(-k^2)K(x) \\
Y_1(x) &= 1 - \tau[a^2(x) + F(x)] + \frac{\tau^2}{2}[a^2(x) + F(x)]^2 - \frac{\tau^3}{6}[a^2(x)F^2(x) + F^2(x)a^2(x) \\
&\quad + F(x)a^2(x)F(x) + F^3(x)] + \frac{\tau^4}{4!}F^4(x)
\end{aligned}$$

$$\begin{aligned}
Y_2(x) &= \frac{\tau^2}{2}a^2(x) - \frac{\tau^3}{6}[2a^4(x) + a^\mu(x)a^2(x)a_\mu(x) + F(x)a^2(x) + a^2(x)F(x) + a^\mu F(x)a_\mu(x)] \\
&\quad - \frac{\tau}{4}[F^2(x)a^2(x) + a^2(x)F^2(x) + a^\mu(x)F^2(x)a_\mu(x) + F(x)a^2(x)F(x) \\
&\quad + F(x)a^\mu(x)F(x)a_\mu(x) + a^\mu(x)F(x)a_\mu(x)F(x)] \\
Y_3(x) &= \frac{\tau^4}{4!}[a^4(x) + a^\mu(x)a^\nu(x)a_\mu(x)a_\nu(x) + a^\mu(x)a^2(x)a_\mu(x)]
\end{aligned}$$

APPENDIX C: $\overline{\mathcal{C}}$ AND $\tilde{\mathcal{C}}$ COEFFICIENTS

$\overline{\mathcal{C}}$ coefficients are defined as

$$\begin{aligned}
\overline{\mathcal{C}}_2 &= 2 \int d\tilde{k} \overline{A}_k [2X_k^3 + 2\frac{X_k^2}{\Lambda^2} + \frac{X_k}{\Lambda^4}] \\
\overline{\mathcal{C}}_3 &= \int d\tilde{k} [\overline{B}_k (2X_k^3 + 2\frac{X_k^2}{\Lambda^2} + \frac{X_k}{\Lambda^4}) + X_k^2 + \frac{X_k}{\Lambda^2}] \\
\overline{\mathcal{C}}_9 &= - \int d\tilde{k} \overline{\mathcal{C}}_k [\frac{X_k}{\Lambda^2} + X_k^2] \\
\overline{\mathcal{C}}_{10} &= 4i \int d\tilde{k} \overline{F}_k [2X_k^3 + 2\frac{X_k^2}{\Lambda^2} + \frac{X_k}{\Lambda^4}] \\
\overline{\mathcal{C}}_{11} &= 4 \int d\tilde{k} \Sigma_k [\frac{X_k}{\Lambda^2} + X_k^2]
\end{aligned} \tag{C1}$$

with

$$\begin{aligned}
\overline{A}_k &= \frac{1}{6}[2\Sigma_k^2 - k^2\Sigma_k\Sigma'_k] \\
\overline{B}_k &= \frac{1}{3}[k^2 + \Sigma_k^2 + k^2\Sigma_k\Sigma'_k] \\
\overline{\mathcal{C}}_k &= \frac{1}{3}[1 - \Sigma_k\Sigma'_k + \frac{3}{2}k^2\Sigma_k'^2] \\
\overline{F}_k &= \frac{1}{6}[-4\Sigma_k^2 + 5k^2\Sigma_k\Sigma'_k + k^2]
\end{aligned} \tag{C2}$$

$\tilde{\mathcal{C}}$ coefficients are defined as

$$\begin{aligned}
\tilde{\mathcal{C}}_2 &= 2 \int d\tilde{k} [\tilde{A}_k (2X_k^3 + 2\frac{X_k^2}{\Lambda^2} + \frac{X_k}{\Lambda^4}) + \frac{k^2}{2}\Sigma_k'^2(\frac{X_k}{\Lambda^2} + X_k^2)] \\
\tilde{\mathcal{C}}_3 &= \int d\tilde{k} [\tilde{B}_k (2X_k^3 + 2\frac{X_k^2}{\Lambda^2} + \frac{X_k}{\Lambda^4}) - (1 - \frac{1}{2}k^2\Sigma_k'^2)(\frac{X_k}{\Lambda^2} + X_k^2)] \\
\tilde{\mathcal{C}}_9 &= \int d\tilde{k} [\frac{1}{3}(k^2\Sigma_k' - \Sigma_k)\Sigma_k''X_k + \tilde{D}_k(\frac{X_k^2}{\Lambda^2} + X_k^2) + \tilde{E}_k(2X_k^3 + 2\frac{X_k^2}{\Lambda^2} + \frac{X_k}{\Lambda^4})] \\
\tilde{\mathcal{C}}_{10} &= 4i \int d\tilde{k} [\tilde{F}_k (2X_k^3 + 2\frac{X_k^2}{\Lambda^2} + \frac{X_k}{\Lambda^4}) + \frac{k^2}{2}\Sigma_k'^2(\frac{X_k}{\Lambda^2} + X_k^2)] \\
\tilde{\mathcal{C}}_{11} &= -2 \int d\tilde{k} k^2\Sigma_k'(\frac{X_k}{\Lambda^2} + X_k^2)
\end{aligned} \tag{C3}$$

with

$$\begin{aligned}
\tilde{A}_k &= \frac{1}{3}(-2k^2\Sigma'_k + \Sigma_k)\Sigma_k(1 + 2\Sigma_k\Sigma'_k) + \frac{1}{6}k^2(2\Sigma_k^2 + k^2)(\Sigma'^2 + \Sigma\Sigma'') - \frac{1}{3}\Sigma_k^2 + \frac{1}{6}k^2\Sigma_k\Sigma'_k \\
\tilde{B}_k &= \frac{1}{3}\Sigma_k(-2k^2\Sigma'_k + \Sigma_k)(1 + 2\Sigma_k\Sigma'_k) + \frac{k^2}{18}(6\Sigma_k^2 + k^2)(\Sigma'^2 + \Sigma_k\Sigma''_k) \\
&\quad - \frac{k^2}{6}(1 + 2\Sigma_k\Sigma'_k) - \frac{1}{3}(k^2 + \Sigma_k^2 + k^2\Sigma_k\Sigma'_k) \\
\tilde{D}_k &= -\frac{1}{2}k^2\Sigma'^2 + k^2(-\frac{1}{6}\Sigma_k\Sigma'_k + \frac{2}{9}k^2\Sigma'_k\Sigma''_k)(1 + 2\Sigma_k\Sigma'_k) + k^2\Sigma'_k(\frac{2}{9}k^2\Sigma'_k - \frac{1}{3}\Sigma_k)(\Sigma'^2_k + \Sigma_k\Sigma'_k) \\
\tilde{E}_k &= k^2\Sigma'_k(\frac{1}{6}\Sigma_k - \frac{1}{9}k^2\Sigma'_k)(1 + 2\Sigma_k\Sigma'_k)^2 \\
\tilde{F}_k &= \Sigma_k\Sigma'_k(\frac{1}{2}k^2 - \frac{4}{3}k^2\Sigma_k\Sigma'_k + \frac{2}{3}\Sigma_k^2) + k^2(\frac{1}{3}\Sigma_k^2 + \frac{1}{9}k^2)(\Sigma'^2_k + \Sigma_k\Sigma''_k) - \frac{k^2}{3}(1 + 2\Sigma_k\Sigma'_k) + \frac{1}{3}k^2]
\end{aligned} \tag{C4}$$